

1 Setup

1.1 Import required packages

```
[2]: import struct
import random
import itertools
import datetime
import multiprocessing
from collections import defaultdict
import json
import traceback
import glob

from matplotlib import pyplot as plt
import matplotlib
from matplotlib import ticker
import numpy as np
from scipy import signal as sig
from scipy import fftpack as fftpack
import ipywidgets
from IPython.display import set_matplotlib_formats

from tqdm.notebook import tqdm
import colorednoise

np.set_printoptions(linewidth=240)
```

```
[3]: %matplotlib widget
%matplotlib inline
set_matplotlib_formats('png', 'pdf')
font = {'family' : 'normal',
        'weight' : 'normal',
        'size'   : 6}
matplotlib.rc('font', **font)
```

1.2 Define mains frequency sampling rate

This is the rate of mains frequency measurements, also called “reporting rate”.

```
[4]: sampling_rate = 10 # sp/s
```

1.3 Library functions

1.3.1 Gold code generator

```
[5]: # From https://github.com/mubeta06/python/blob/master/signal_processing/sp/gold.py
preferred_pairs = {5:[[2],[1,2,3]], 6:[[5],[1,4,5]], 7:[[4],[4,5,6]],
                     8:[[1,2,3,6,7],[1,2,7]], 9:[[5],[3,5,6]],
                     10:[[2,5,9],[3,4,6,8,9]], 11:[[9],[3,6,9]]}

def gen_gold(seq1, seq2):
    gold = [seq1, seq2]
    for shift in range(len(seq1)):
        gold.append(seq1 ^ np.roll(seq2, -shift))
    return gold

def gold(n):
    n = int(n)
    if not n in preferred_pairs:
        raise KeyError('preferred pairs for %s bits unknown' % str(n))
    t0, t1 = preferred_pairs[n]
    (seq0, _st0), (seq1, _st1) = sig.max_len_seq(n, taps=t0), sig.
    ↪max_len_seq(n, taps=t1)
    return gen_gold(seq0, seq1)
```

1.3.2 Gold code modulator

```
[6]: def modulate(data, nbits=5, pad=True):
    # 0, 1 -> -1, 1
    mask = np.array(gold(nbits))*2 - 1

    sel = mask[data>>1]
    data_lsb_centered = ((data&1)*2 - 1)

    signal = (np.multiply(sel, np.tile(data_lsb_centered, (2**nbits-1, 1)).T).
    ↪flatten() + 1) // 2
    if pad:
        return np.hstack([ np.zeros(len(mask)), signal, np.zeros(len(mask)) ])
    else:
        return signal
```

1.3.3 Gold code correlator

This function, used by the prototype demodulation algorithm below, correlates a signal against all 2^n+1 Gold sequences. Given an input signal of length k it produces an output matrix of dimensions $(2^n + 1, k)$ with one column for each shift of the reference Gold sequences w.r.t. the input signal and one row per Gold sequence.

```
[7]: def correlate(sequence, nbits=5, decimation=1, mask_filter=lambda x: x):
    mask = np.tile(np.array(gold(nbits))[:, :, np.newaxis]*2 - 1, (1, 1, ↴decimation)).reshape((2**nbits + 1, (2**nbits-1) * decimation))

    # Our input signal has large DC bias. Remove DC bias to reduce numerical ↴errors during correlation.
    sequence -= np.mean(sequence)

    return np.array([np.correlate(sequence, row, mode='full') for row in mask])
```

1.3.4 Read recorded mains frequency data from exported capture file

```
[8]: with open('data/fmeas_export_ocxo_2day.bin', 'rb') as f:
    meas_data = np.copy(np.frombuffer(f.read(), dtype='float32'))
    print('mean:', np.mean(meas_data), 'len:', len(meas_data))
    meas_data -= np.mean(meas_data)
    def mains_noise(n):
        last_valid = len(meas_data) - n
        start = np.random.randint(last_valid)
        return meas_data[start:start+n]
```

mean: 50.00341 len: 1946174

1.3.5 Test signal generator

This generates deterministically random test data, modulates it using the Gold code modulator, scales it to a given target amplitude and adds noise from recorded data above.

```
[9]: def generate_test_signal(duration, nbits=6, signal_amplitude=2.0e-3, ↴decimation=10, seed=0, data=None):
    test_data = np.random.RandomState(seed=seed).randint(0, 2 * (2**nbits), ↴duration) if data is None else data

    signal = np.repeat(modulate(test_data, nbits) * 2.0 - 1, decimation) * ↴signal_amplitude
    noise = mains_noise(len(signal))

    return test_data, signal + noise
```

1.4 Signal exporters for hardware testing

The following two functions generate test data to test the firmware implementation in software simulations.

```
[10]: def do_export_clean():
    test_duration = 200
    test_nbts = 5
```

```

test_signal_amplitude=2.0e-3
test_decimation=10

for test_signal_amplitude in [2.0e-3, 20e-3, 200e-3, 2]:
    test_data = np.random.RandomState(seed=0).randint(0, 2 * ↴
    ↵(2**test_nbites), test_duration)
        #test_data = np.array([0, 1, 2, 3] * 50)
        signal = np.repeat(modulate(test_data, test_nbites, pad=False) * 2.0 - ↴
    ↵1, test_decimation) * test_signal_amplitude
        with open(f'dsss_test_signals/ ↵
    ↵dsss_test_noiseless_{test_signal_amplitude*1000:.0f}mHz.bin', 'wb') as f:
            for e in signal:
                f.write(struct.pack('<f', e))

```

```
[11]: def do_export_noisy():
    test_duration = 32
    test_nbites = 5
    test_signal_amplitude=2.0e-3
    test_decimation=10
    test_signal_amplitude = 200e-3
    noise_level = 10e-3

    #test_data = np.random.RandomState(seed=0).randint(0, 2 * (2**test_nbites), ↴
    ↵test_duration)
        #test_data = np.array([0, 1, 2, 3] * 50)
        test_data = np.array(range(test_duration))
        signal = np.repeat(modulate(test_data, test_nbites, pad=False) * 2.0 - 1, ↴
    ↵test_decimation) * test_signal_amplitude
        noise = colorednoise.powerlaw_psd_gaussian(1, len(signal)*10) * noise_level
        noise[-int(1.5*len(signal)):] [:len(signal)] += signal

        with open(f'dsss_test_signals/dsss_test_noisy_padded.bin', 'wb') as f:
            for e in noise:
                f.write(struct.pack('<f', e))

```

2 The algorithm

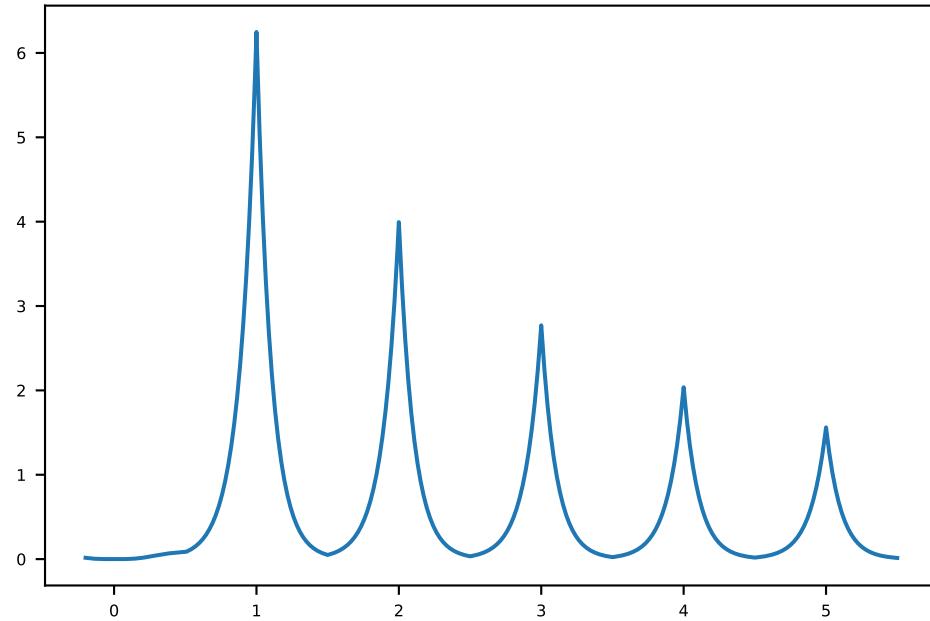
2.1 First we define some components used in our algorithm.

The following function is used to score a new correlation peak against previous peaks. The aim is to assign a high fitness the closer the peak lies to a multiple of one symbol period from the last peak. The first peak is the ideal case, subsequent peaks correspond to dropped symbols.

```
[12]: nonlinear_distance = lambda x: 100**((2*np.abs(0.5-x%1)) / (np.abs(x)+3)**2 * ↴
    ↵(np.clip(np.abs(x), 0, 0.5) * 2)**5
```

```
def plot_distance_func():
    fig, ax = plt.subplots()
    x = np.linspace(-0.2, 5.5, 10000)
    ax.plot(x, nonlinear_distance(x))
```

```
[13]: plot_distance_func()
```



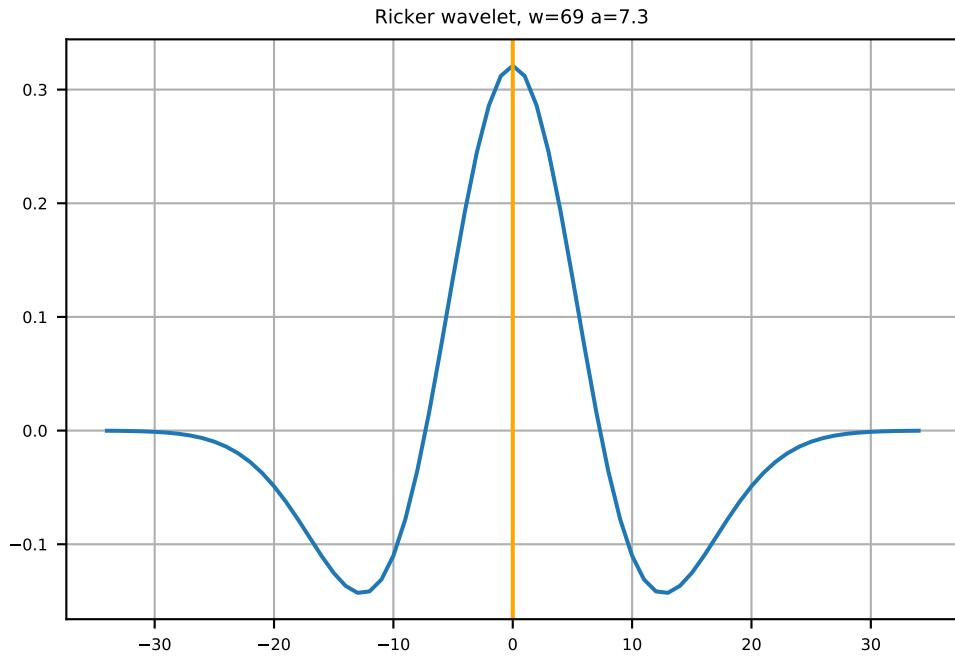
2.2 Ricker wavelet computations for firmware implementation

For our firmware implementation we need a ricker wavelet lookup table. To find out the size of this lookup table, we calculate the truncation error for a given size below.

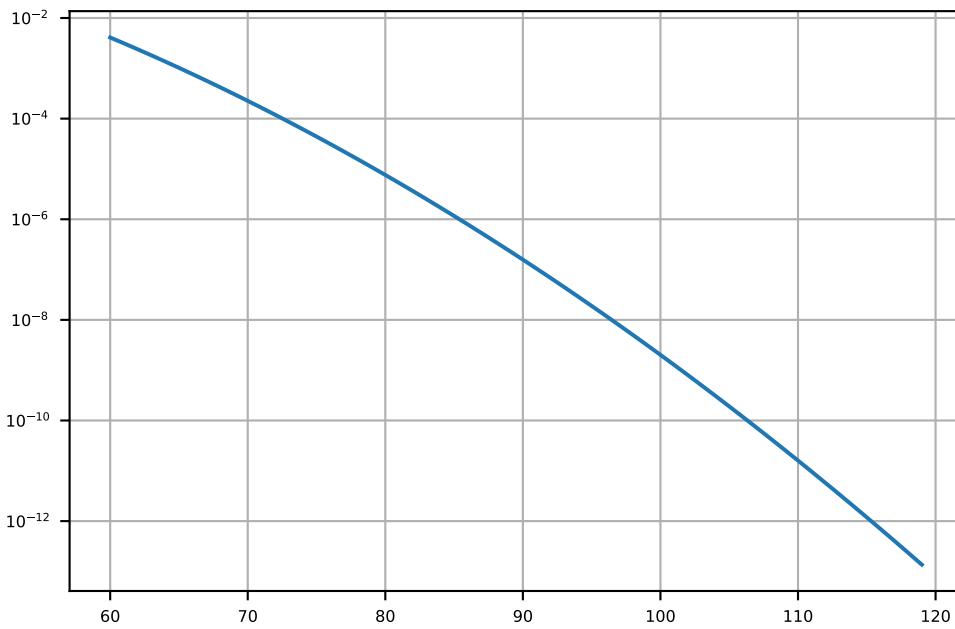
```
[14]: noprint = lambda *args, **kwargs: None
```

```
[15]: fig, ax = plt.subplots()
w = 69
a = 7.3
ax.plot(range(-w//2+1, w//2+1), sig.ricker(w, a))
ax.grid()
ax.axvline(0, color='orange')
ax.set_title(f'Ricker wavelet, w={w} a={a}')
```

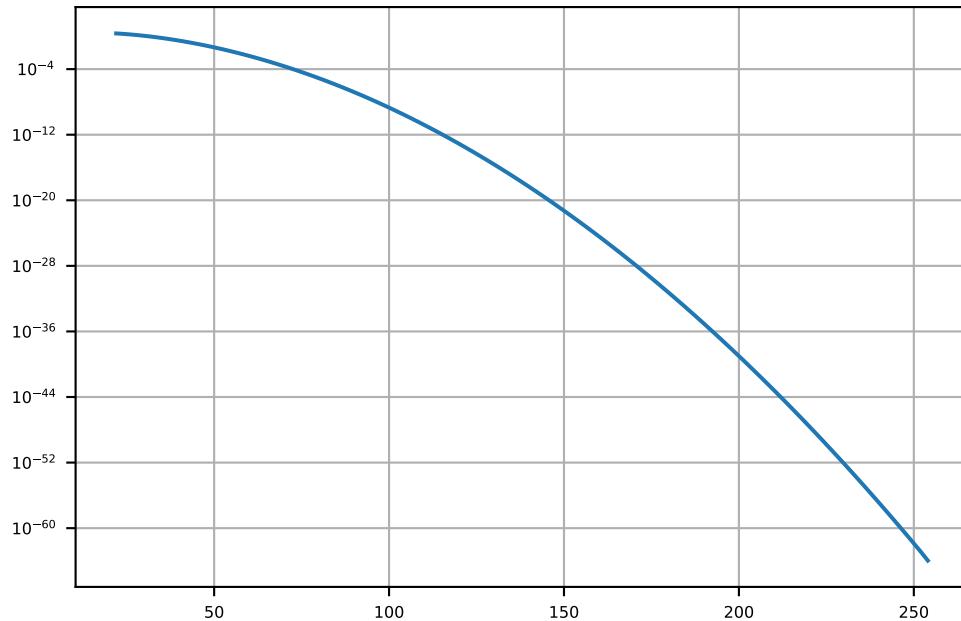
```
[15]: Text(0.5, 1.0, 'Ricker wavelet, w=69 a=7.3')
```



```
[16]: fig, ax = plt.subplots()
r = list(range(60, 120))
ax.plot(r, [sum(sig.ricker(w, a)) for w in r])
ax.set_yscale('log')
ax.grid()
```



```
[17]: fig, ax = plt.subplots()
sw = 256
w = sig.ricker(sw, a)
r = list(range(1, sw//2 - 10))
d = [-sum(w[:i]) - sum(w[-i:]) for i in r]
ax.plot([sw-2*x for x in r], d)
ax.set_yscale('log')
ax.grid()
```



2.3 Demodulation algorithm and testing function

The following function contains our prototype demodulation algorithm implementation along with test code applying it to simulated input data. By repeatedly running this function while sweeping parameters we can create plots of our algorithm's performance under various conditions.

```
[18]: def run_ser_test(sample_duration=128, nbits=6, signal_amplitude=2.0e-3,
→decimation=10, threshold_factor=4.0, power_avg_width=2.5, max_lookahead=6.5,
→pol_score_factor=1.0, seed=0, ax=None, print=print, ser_maxshift=3,
→debug_range=None):

    # Generate test data for this simulation run
    test_data, signal = generate_test_signal(sample_duration, nbits,
→signal_amplitude, decimation, seed)

    # === Begin of our prototype demodulation algorithm. ===
```

```

# (1) Correlate the input signal against all  $2^{n+1}$  gold sequences using the
# correlation function defined above.
cor_an = correlate(signal, nbits=nbits, decimation=decimation)

# span to compute average power measurements for peak finding over, in
# samples
power_avg_width = int(power_avg_width * (2**nbits - 1) * decimation)

bit_period = (2**nbits) * decimation # duration of one DSSS symbol
peak_group_threshold = 0.05 * bit_period # Duration over which to group
# several detected peaks into one, in samples
hole_patching_threshold = 0.01 * bit_period # Duration over which to ignore
# temporary dropouts in signal during grouping

# (2) Calculate continuous wavelet transform of correlator output and a
# ricker wavelet. The width factor of the ricker wavelet at 0.73*decimation was
# determined empirically. This transform acts like a sharpening filter on
# our peaks and will increase SNR for the following thresholding step.
cwt_res = np.array([ sig.cwt(row, sig.ricker, [0.73 * decimation]).
# flatten() for row in cor_an ])
if ax:
    ax.grid()
    ax.plot(cwt_res.T)

# (3) Threshold CWT'ed correlator outputs using the factors defined above.
# Classify any sample as part of a peak that is threshold_factor times
# larger than the average of the surrounding signal.
th = np.array([ np.convolve(np.abs(row), np.ones((power_avg_width,))/
# power_avg_width, mode='same') for row in cwt_res ])

# Helper function for thresholding
def compare_th(elem):
    idx, (th, val) = elem
    #print('compare_th:', th.shape, val.shape)
    return np.any(np.abs(val) > th*threshold_factor)

# (4) Group samples above threshold value into spans
peaks = [ list(group) for val, group in itertools.groupby(enumerate(zip(th.
#T, cwt_res.T)), compare_th) if val ]
peaks_processed = []
peak_group = []
# For each span of samples above threshold, try to coalesce this span with
# adjacent spans if it is close enough
for group in peaks:
    pos = np.mean([idx for idx, _val in group])

```

```

        #pol = np.mean([max(val.min(), val.max(), key=abs) for _idx, (_th, val) in group])
        pol = max([max(val.min(), val.max(), key=abs) for _idx, (_th, val) in group], key=abs)
        pol_idx = np.argmax(np.bincount([np.argmax(np.abs(val)) for _idx, (_th, val) in group]))
        peaks_processed.append((pos, pol, pol_idx))
        #print(f'group', pos, pol, pol_idx)
        #for pol, (_idx, (_th, val)) in zip([max(val.min(), val.max(), key=abs) for _idx, (_th, val) in group], group):
        if peak_group:
            msg += f'continuing previous group: {peak_group[-1]},'
            group_start, last_pos, last_pol, peak_pos, last_pol_idx = peak_group[-1]
        if abs(pol) > abs(last_pol):
            msg += 'larger, '
            if ax:
                ax.axvline(pos, color='magenta', alpha=0.3)
            peak_group[-1] = (group_start, pos, pol, pos, pol_idx)

        else:
            msg += 'smaller, '
            if ax:
                ax.axvline(pos, color='blue', alpha=0.5)
            peak_group[-1] = (group_start, pos, last_pol, peak_pos, last_pol_idx)
        else:
            last_pos = None

        if not peak_group or pos - last_pos > peak_group_threshold:
            msg += 'terminating, '
            if peak_group:
                msg += f'previous group: {peak_group[-1]},'
                peak_pos = peak_group[-1][3]
                if ax:
                    ax.axvline(peak_pos, color='red', alpha=0.6)
                    #ax3.text(peak_pos-20, 2.0, f'{0 if pol < 0 else 1}', horizontalalignment='right', verticalalignment='center', color='black')
            msg += f'new group: {(pos, pos, pol, pos, pol_idx)} '

```

```

peak_group.append((pos, pos, pol, pos, pol_idx))
if ax:
    ax.axvline(pos, color='cyan', alpha=0.5)

if debug_range:
    low, high = debug_range
    if low < pos < high:
        print(msg)
        print(group)

# Calculate average magnitude of all found peaks for normalization in next step
avg_peak = np.mean(np.abs(np.array([last_pol for _1, _2, last_pol, _3, _4 in peak_group])))
print('avg_peak', avg_peak)

# (5) Perform Maximum likelihood estimation to group peaks into chains of peaks spaced at one-symbol duration intervals.
noprint = lambda *args, **kwargs: None
def mle_decode(peak_groups, print=noprint):
    """ Maximum likelihood estimation decoding.

    This function tries to find sequences of peaks that are spaced at one-symbol intervals. It will yield candidate sequences along with their by-fitness.

    A sequence is evaluated better the higher its peaks, the closer they match one-symbol intervals from each other and the longer it is.
    """
    # For each peak, extract index inside capture (in samples), polarity and the index of the Gold sequence that matched to produce this peak
    peak_groups = [ (pos, pol, idx) for _1, _2, pol, pos, idx in peak_groups ]

    # Initially populate candidate array with all peaks in first couple of symbol periods from signal start.
    candidates = [ (abs(pol)/avg_peak, [(pos, pol, idx)]) for pos, pol, idx in peak_groups if pos < bit_period*2.5 ]

    # Iterate while there are candidates remaining
    while candidates:
        chain_candidates = [] # candidates for next iteration
        for chain_score, chain in candidates:
            pos, ampl, _idx = chain[-1]
            score_fun = lambda pos, npos, npol: pol_score_factor*abs(npol)/avg_peak + nonlinear_distance((npos-pos)/bit_period)

```

```

# For this candidate, consider all peaks that might extend it up
→to a longer chain up to some maximum timestamp in the future.
    next_candidates = sorted([ (score_fun(pos, npos, npol), npos, npol, nidx)
→for npos, npol, nidx in peak_groups if pos < npos < pos + bit_period*max_lookahead ], reverse=True)

        print(f'    candidates for {pos}, {ampl}:')
        for score, npos, npol, nidx in next_candidates:
            print(f'        {score:.4f} {npos:.2f} {npol:.2f} {nidx:.2f}')

    nch, cor_len = cor_an.shape
    if cor_len - pos < 1.5*bit_period or not next_candidates:
        # If we have hit the end of our signal or if we did not
→detect any more peaks, calculate this candidate's score and yield it to the
→caller.
        score = sum(score_fun(opos, npos, npol) for opos, _opol,
→_oidx), (npos, npol, _nidx) in zip(chain[:-1], chain[1:])) / len(chain)
        yield score, chain

    else:
        # If we have not yet hit the end of our signal, and we
→still have peaks left try to extend the current candidate with each of these
→peaks in turn.
        # Calculate the score of the resulting extended chains and
→if they are not too bad, append them to the candidates for the next iteration
        print('extending')
        for score, npos, npol, nidx in next_candidates[:3]:
            if score > 0.5:
                new_chain_score = chain_score * 0.9 + score * 0.1
                chain_candidates.append((new_chain_score, chain +
→[(npos, npol, nidx)]))

    # For next iteration select top-n highest scoring candidates just
→computed
    print('chain candidates:')
    for score, chain in sorted(chain_candidates, reverse=True):
        print('        ', [(score, [(f'{pos:.2f}', f'{pol:.2f}') for pos, pol, _idx in chain])])
    candidates = [ (chain_score, chain) for chain_score, chain in
→sorted(chain_candidates, reverse=True)[:10] ]

# Group peaks into chains and select the chain with the highest score
res = sorted(mle_decode(peak_group, print=noprint), reverse=True)
#for i, (score, chain) in enumerate(res):
#    print(f'Chain {i}@{score:.4f}: {chain}')

```

```

(_score, chain), *_ = res

def viz(chain, peaks):
    last_pos = None
    for pos, pol, nidx in chain:
        if last_pos:
            delta = int(round((pos - last_pos) / bit_period))
            if delta > 1:
                print(f'skipped {delta-1} symbols at {pos}/{last_pos}')

        # Hole patching routine
        for i in range(1, delta):
            est_pos = last_pos + (pos - last_pos) / delta * i

            icandidates = [ (ipos, ipol, iidx) for ipos, ipol, iidx in
                           peaks if abs(est_pos - ipos) < hole_patch_threshold ]
            if not icandidates:
                yield None
                continue

            ipos, ipol, iidx = max(icandidates, key = lambda e:
                                   abs(e[1]))

            decoded = iidx*2 + (0 if ipol < 0 else 1)
            print(f'interpolating, last_pos={last_pos},',
                  f'delta={delta}, pos={pos}, est={est_pos} dec={decoded}')
            yield decoded

        decoded = nidx*2 + (0 if pol < 0 else 1)
        yield decoded
        if ax:
            ax.axvline(pos, color='blue', alpha=0.5)
            ax.text(pos-20, 0.0, f'{decoded}', horizontalalignment='right',
                    verticalalignment='center', color='black')

    last_pos = pos

decoded = list(viz(chain, peaks_processed))
print('decoding [ref|dec]:')
match_result = []
for shift in range(-ser_maxshift, ser_maxshift):
    msg = f'==== shift = {shift} ===\n'
    failures = -shift if shift < 0 else 0 # we're skipping the first $shift
    for symbols
        a = test_data if shift > 0 else test_data[-shift:]
        b = decoded if shift < 0 else decoded[shift:]
        for i, (ref, found) in enumerate(itertools.zip_longest(a, b)):
```

```

        if ref is None: # end of signal
            break
        msg += f'{ref if ref is not None else -1:>3d}|{found if found is not None else -1:>3d} {" " if ref==found else " "}" '
        if ref != found:
            failures += 1
        if i%8 == 7:
            msg += '\n'
    match_result.append((failures, msg))
failures, msg = min(match_result, key=lambda e: e[0])
print(msg)
ser = failures/len(test_data)
print(f'Symbol error rate e={ser}: {failures}/{len(test_data)}')
br = sampling_rate / decimation / (2**nbits) * nbits * (1 - ser) * 3600
print(f'maximum bitrate r={br} b/h')
return ser, br

```

```
[19]: default_params = dict(
    decimation=10,
    power_avg_width=2.5,
    max_lookahead=6.5)

fig, ax = plt.subplots(figsize=(12, 9))

def calculate_ser(v, seed, nbits, thf, reps, duration):
    st = np.random.RandomState(seed)
    params = dict(default_params)
    params['signal_amplitude'] = v
    params['nbits'] = nbits
    params['threshold_factor'] = thf
    sers, brs = [], []
    for i in range(reps):
        seed = st.randint(0xffffffff)
        try:
            ser, br = run_ser_test(**params, sample_duration=duration, print=noprint, seed=seed)
            sers.append(ser)
            brs.append(br)
        except Exception as e:
            traceback.print_exc()
            print('got', e, 'seed', seed, 'params', params)
            #sers.append(1.0)
            #brs.append(0.0)
    #print(f'nbits={nbits} ampl={v:.5f} seed={seed:08x} > ser={ser:.5f}')
    sers, brs = np.array(sers), np.array(brs)
    ser, std = np.mean(sers), np.std(sers)
```

```

#print(f'signal_amplitude={v:.5f}: ser={ser:.5f} ±{std:.5f}, br={np.
˓→mean(brs):.5f}')
    return ser, std

results = []
with tqdm(total = 0) as tq:
    with multiprocessing.Pool(multiprocessing.cpu_count()//2) as pool:
        for nbits, thf, reps, points, duration in [(5, 4.0, 5, 25, 64), (6, 4.
˓→0, 5, 25, 64)]: #[(5, 4.0, 50, 25, 128), (6, 4.0, 25, 25, 64), (7, 5.0, 10,
˓→10, 64), (8, 6.0, 5, 10, 32)]:
            #print(f'nbits={nbits}')
            st = np.random.RandomState(0)
            vs = 0.1e-3 * 10 ** np.linspace(0, 1.5, points)
            results[nbits] = [pool.apply_async(calculate_ser, (v, st.
˓→randint(0xffffffff), nbits, thf, reps, duration), callback=lambda _res: tq.
˓→update(1)) for v in vs]
            tq.total += len(vs)
            tq.refresh()

pool.close()
pool.join()

print(f'scheduled {tq.total} tasks. waiting...')
results = { nbits: [res.get() for res in series] for nbts, series in
˓→results.items() }
print('done')

with open(f'dsss_experiments_res-{datetime.datetime.now():%Y-%m-%d %H:%M:%S}.
˓→json', 'w') as f:
    json.dump(results, f)

for nbts, res in results.items():
    data = np.array(res)
    sers, stds = data[:,0], data[:,1]

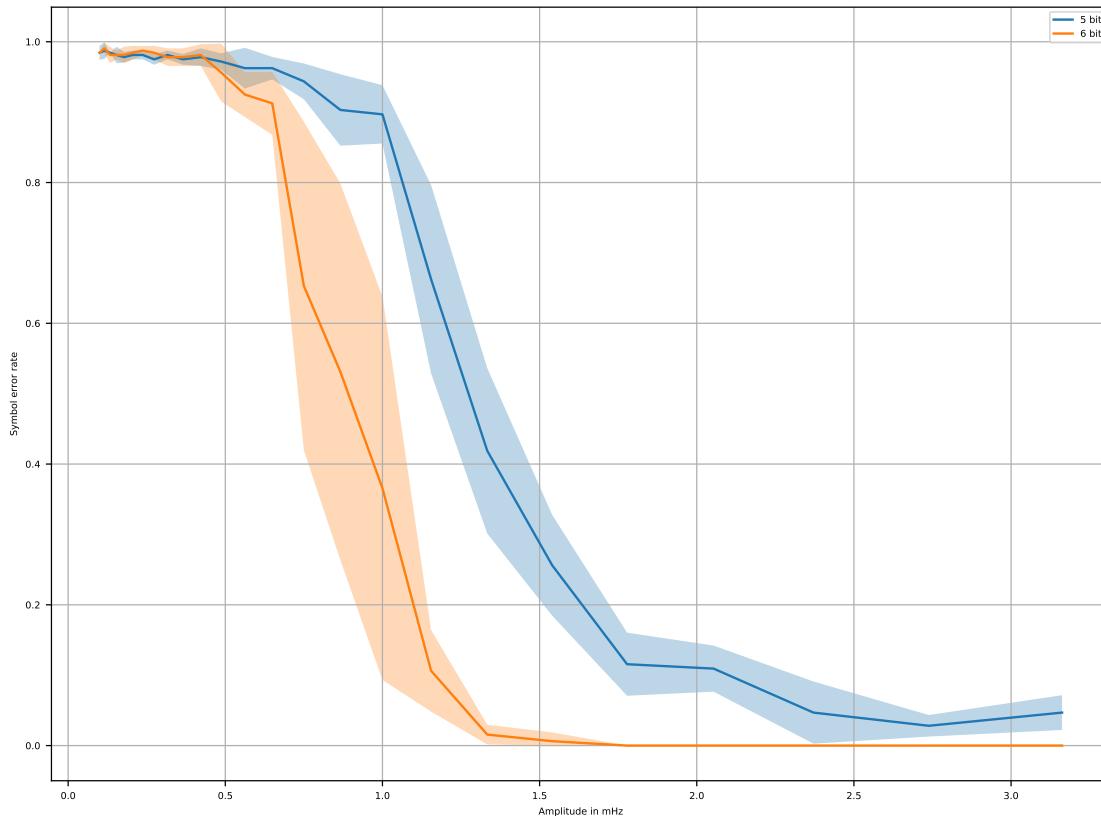
    l, = ax.plot(vs*1e3, np.clip(sers, 0, 1), label=f'{nbts} bit')
    ax.fill_between(vs*1e3, np.clip(sers + stds, 0, 1), np.clip(sers - stds, 0,
˓→1), facecolor=l.get_color(), alpha=0.3)
ax.grid()
ax.set_xlabel('Amplitude in mHz')
ax.set_ylabel('Symbol error rate')
ax.legend()

```

got not enough values to unpack (expected at least 1, got 0) seed 2919184728
params {'decimation': 10, 'power_avg_width': 2.5, 'max_lookahead': 6.5,
'signal_amplitude': 0.0031622776601683794, 'nbts': 5, 'threshold_factor': 4.0}

```
got not enough values to unpack (expected at least 1, got 0) seed 2642033202
params {'decimation': 10, 'power_avg_width': 2.5, 'max_lookahead': 6.5,
'signal_amplitude': 0.0027384196342643613, 'nbits': 6, 'threshold_factor': 4.0}
scheduled 50 tasks. waiting...
done
```

```
[19]: <matplotlib.legend.Legend at 0x7f5bd3fb75b0>
```



```
[30]: fig, ax = plt.subplots(figsize=(3, 2))

#     sers, brs = np.array(sers), np.array(brs)
#     ser, std = np.mean(sers), np.std(sers)
#     results = { nbts: [ res.get() for res in series ] for nbts, series in
#     ↪results.items() }

with open(f'data/dsss_experiments_res-2020-02-19-19-30-05.json', 'r') as f:
    results = json.load(f)

for nbts, series in results.items():
```

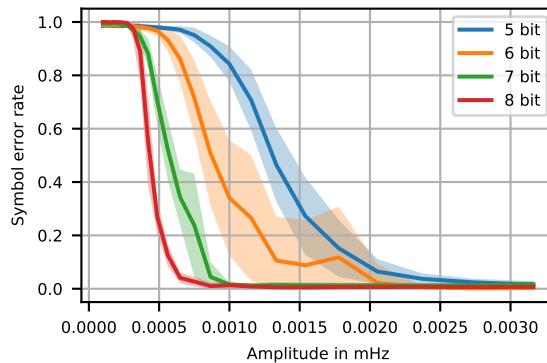
```

    series = [ [ mean for mean, _std, _msg in reps if mean is not None ] for
    ↪reps in series ]
    sers = np.array([ np.mean(values) for values in series ])
    stds = np.array([ np.std(values) for values in series ])

    # FIXME HACK HACK HACK
    vs = 0.1e-3 * 10 ** np.linspace(0, 1.5, 25)

    l, = ax.plot(vs, np.clip(sers, 0, 1), label=f'{nbits} bit')
    ax.fill_between(vs, np.clip(sers + stds, 0, 1), np.clip(sers - stds, 0, 1), ↪
    ↪facecolor=l.get_color(), alpha=0.3)
ax.grid()
ax.set_xlabel('Amplitude in mHz')
ax.set_ylabel('Symbol error rate')
ax.legend()
fig.tight_layout()
fig.savefig('fig_out/dsss_gold_nbites_overview.pdf')

```



```

[29]: default_files = [
    #      'data/dsss_experiments_res-2020-02-20-12-18-35.json',
    #      'data/dsss_experiments_res-2020-02-20-12-26-07.json',
    #      'data/dsss_experiments_res-2020-02-20-12-29-02.json'
    'data/dsss_experiments_res-par107-run115-0-2020-04-07-11-41-37.json',
    'data/dsss_experiments_res-par107-run115-1-2020-04-07-13-23-42.json',
    'data/dsss_experiments_res-par107-run115-2-2020-04-07-08-57-38.json',
    'data/dsss_experiments_res-par107-run115-3-2020-04-07-15-48-04.json',
]

def load_results(*files):
    results = []
    for fn in files:
        with open(fn, 'r') as f:
            results += json.load(f)

```

```

    return results

def thf_dependence_plot(results, plot_nbts=6,
                       ax=None, cbar_ax=None, intercept_ax=None,
                       xlabel=True, x2label=False, ylabel=True, y2label=True, u
→y2ticks=True, ic_ylim=[-0.5, 5],
                       legend_loc=4, split_legend=False, calc_best_ampl=False):

    thfs = [thf for (_nbts, thf, _reps, _points, _duration, _decimation), u
→series in results]
    cmap = matplotlib.cm.viridis
    cm_func = lambda x: cmap((x - min(thfs)) / (max(thfs) - min(thfs)))

    thf_sers = {}
    for (nbts, thf, reps, points, duration, decimation), series in results:
        if nbts != plot_nbts:
            continue
        data = [ [ mean for mean, _std, _msg in reps if mean is not None ] for u
→_amp, reps in series ]
        amps = [ amp*1000 for amp, _reps in series ]
        sers = np.array([ np.mean(values) for values in data ])
        stds = np.array([ np.std(values) for values in data ])
        thf_sers[thf] = list(zip(amps, sers, stds))

        if ax:
            l, = ax.plot(amps, np.clip(sers, 0, 1), label=f'thf={thf}', u
→color=cm_func(thf))
            ax.fill_between(amps, np.clip(sers + stds, 0, 1), np.clip(sers - u
→stds, 0, 1), facecolor=l.get_color(), alpha=0.2)
            ax.axhline(0.5, color='gray', ls=(0, (3, 4)), lw=0.8)

        if ax:
            ax.grid()
            ax.set_title(f'{plot_nbts}-bit Gold code')
            if xlabel:
                ax.set_xlabel('Amplitude [mHz]')
            if ylabel:
                ax.set_ylabel('Symbol Error Rate')

    def plot_base_amp(ax):
        base_sers = {}
        for thf, sers in thf_sers.items():
            base = np.mean([ser for amp, ser, std in sorted(sers)[-2:]])
            base_std = np.sqrt(np.mean([std**2 for amp, ser, std in u
→sorted(sers)[-2:]]))
            base_sers[thf] = (base, base_std)


```

```

x = sorted(base_sers.keys())
y = np.array([ base_sers[thf][0] for thf in x ])
std = np.array([ base_sers[thf][1] for thf in x ])
l = ax.plot(x, y, label='SER at large amplitudes')
ax.fill_between(x, y-std, y+std, color=l[0].get_color(), alpha=0.3)
return l

def plot_intercepts(ax, SER_TH = 0.5):
    intercepts = {}
    for thf, sers in thf_sers.items():
        last_ser, last_amp, last_std = 0, 0, 0
        for amp, ser, std in sorted(sers):
            if last_ser > SER_TH and ser < SER_TH:
                icp = last_amp + (SER_TH - last_ser) / (ser - last_ser) * (amp - last_amp)
                ic_std = abs(last_amp - amp) / 2 # np.sqrt(np.mean(last_std**2 + std**2))
                intercepts[thf] = (icp, ic_std)
                break
            last_amp, last_ser = amp, ser
        else:
            intercepts[thf] = None, None

    ser_valid = [thf for thf, (ser, _std) in intercepts.items() if ser is not None]
    #ax.axvline(min(ser_valid), color='red')
    #ax.axvline(max(ser_valid), color='red')

    x = sorted(intercepts.keys())
    data = np.array([ intercepts[thf] for thf in x ])
    y = data[:,0]
    std = data[:,1]

    if ax:
        ax.set_xlim([min(x), max(x)])
        l = ax.plot(x, y, label='Amplitude at SER=0.5', color='orange')
    else:
        l = None

    x, y, std = zip(*[ (le_x, le_y, le_std) for le_x, le_y, le_std in zip(x, y, std) if le_y is not None ])
    y, std = np.array(y), np.array(std)
    if ax:
        ax.fill_between(x, y-std, y+std, color=l[0].get_color(), alpha=0.3)

    trans = matplotlib.transforms.blended_transform_factory(ax.transData, ax.transAxes)

```

```

        ax.fill_between([-1, min(ser_valid)], 0, 1, facecolor='red', u
→alpha=0.2, transform=trans, zorder=1)
        ax.fill_between([max(ser_valid), max(ser_valid)*10], 0, 1, u
→facecolor='red', alpha=0.2, transform=trans)
        ax.set_ylim([min(y)*0.9, max(y)*1.1])
        ax.grid()

    best_ampl = (np.inf, np.nan)
    for yval, stdval in zip(y, std):
        if yval < best_ampl[0]:
            best_ampl = [yval, stdval]

    return l, best_ampl

if intercept_ax:
    if isinstance(intercept_ax, tuple):
        intercept_ax, intercept_ax_twin = intercept_ax
    else:
        intercept_ax_twin = intercept_ax.twinx()

if intercept_ax or calc_best_ampl:
    l1, best_ampl = plot_intercepts(intercept_ax)
else:
    best_ampl = None

if intercept_ax:
    l2 = plot_base_amp(intercept_ax_twin)

intercept_ax.set_title(f'{plot_nbites}-bit Gold code')
if xlabel:
    intercept_ax.set_xlabel('Threshold factor')
if x2label:
    intercept_ax_twin.set_xlabel('Threshold factor')
if ylabel:
    intercept_ax.set_ylabel('Amplitude [mHz]')
intercept_ax.set_ylim(ic_ylim)
intercept_ax_twin.set_ylim([-0.1, 1])
if y2label:
    intercept_ax_twin.set_ylabel('Symbol Error Rate')
if not y2ticks:
    intercept_ax_twin.set_yticklabels([])
if legend_loc is not None:
    if split_legend:
        intercept_ax.legend(l1, [l1[0].get_label()], loc=legend_loc)
        intercept_ax_twin.legend(l2, [l2[0].get_label()], u
→loc=legend_loc)
    else:

```

```

        intercept_ax.legend(l1 + l2, [l.get_label() for l in l1+l2], u
        ↪loc=legend_loc)

    if cbar_ax:
        norm = matplotlib.colors.Normalize(vmin=min(thfs), vmax=max(thfs))
        cb1 = matplotlib.colorbar.ColorbarBase(cbar_ax, cmap=cmap, norm=norm, u
        ↪orientation='vertical', label="Threshold factor")

    return best_ampl

import warnings
warnings.filterwarnings('ignore')

def plot_gold_sensitivity(results, nbitsets=[5,6,7,8]):
    nbitsets = np.array(nbitsets)
    ampls = np.array([ thf_dependence_plot(plot_nbits=dep_n, results=results, u
    ↪calc_best_ampl=True) for dep_n in nbitsets ])
    fig, ax = plt.subplots(figsize=(3, 2))
    l = ax.plot(nbitsets, ampls[:,0])
    ax.fill_between(nbitsets, ampls[:,0]-ampls[:,1], ampls[:,0]+ampls[:,1], u
    ↪color=l[0].get_color(), alpha=0.3)
    ax.grid()
    ax.set_xlabel('Gold code bits')
    ax.set_ylabel('Amplitude at SER=0.5 [mHz]')
    ax.set_ylim([0, 2])
    ax.xaxis.set_major_locator(ticker.MultipleLocator(1.0))
    fig.tight_layout()
    return fig

def plot_amplitude_ber(results, grid=(2, 3), nbitsets=[5,6,7,8], figsize=(12, u
    ↪9), xlim=None, xlog=False):
    fig = plt.figure(figsize=figsize)
    gs = plt.GridSpec(*grid, figure=fig, width_ratios=[1, 1, 0.05])

    cbar_ax = fig.add_subplot(gs[0, 2])

    axs = np.empty([2, 2], dtype=object)
    for i, nbits in enumerate(nbitsets):
        row, col = i//2, i%2

        ax = axs[row, col] = fig.add_subplot(gs[row, col])
        if xlog:
            ax.set_xscale('log')
        if xlim is not None:
            ax.set_xlim(xlim)
        if row == 1:

```

```

        ax.get_shared_x_axes().join(axes[0, col])
    if col == 1:
        ax.get_shared_y_axes().join(axes[row, 0])

    xlabel = row==1 if len(nbitses) > 2 else True
    thf_dependence_plot(plot_nb_bits=nbits, ax=ax, cbar_ax=cbar_ax if i==0 else None, xlabel=xlabel, ylabel=col==0, results=results)

    return fig

def plot_thf_graph(results, nbitses=[5,6,7,8], ic_ylim=[-0.5, 5], figsize=(12, 9)):
    fig, axes = plt.subplots(2, 2, figsize=figsize, sharex='col', sharey='row', gridspec_kw={'wspace': 0.1})
    for nb_bits, ax, ax_below in zip(nbitses, axes.flatten(), [*axes.flatten()[2:], None, None]):
        if len(nbitses) <= 2:
            ax = ax, ax_below
            ax_below.grid()
            y2label = nb_bits in [5, 7]
            legend_loc = 9
            y2ticks = True
        else:
            y2ticks = y2label = nb_bits in [6, 8]
            legend_loc = 1
        thf_dependence_plot(plot_nb_bits=nb_bits, intercept_ax=ax,
                             xlabel=nb_bits in [7, 8], ylabel=nb_bits in [5, 7], y2label=y2label, ic_ylim=ic_ylim,
                             y2ticks=y2ticks, x2label=len(nbitses) <= 2,
                             legend_loc=legend_loc if nb_bits == nbitses[-1] else None,
                             split_legend = len(nbitses) <= 2,
                             results=results)
    return fig

plot_gold_sensitivity(load_results(*default_files))\
.savefig('fig_out/dsss_gold_nb_bits_sensitivity.pdf');

plot_amplitude_ber(load_results(*default_files), figsize=(7, 4))\
.savefig('fig_out/dsss_thf_amplitude_5678.pdf');

plot_thf_graph(load_results(*default_files), figsize=(7, 4))\
.savefig('fig_out/dsss_thf_sensitivity_5678.pdf')

# Note: due to a mistake these "par114" files actually contain "par115" data.

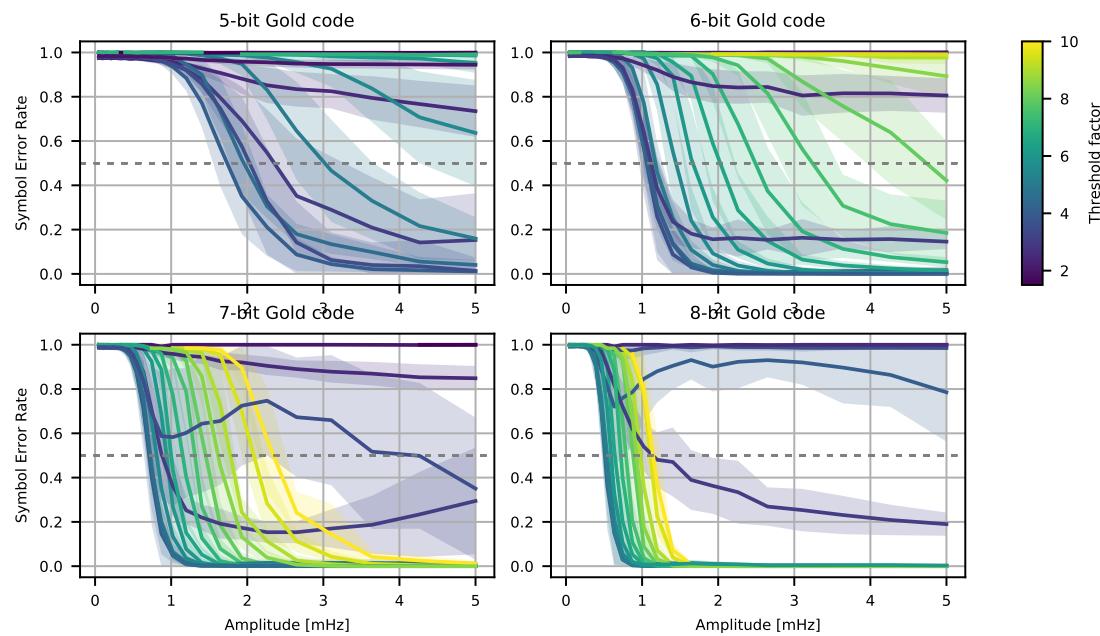
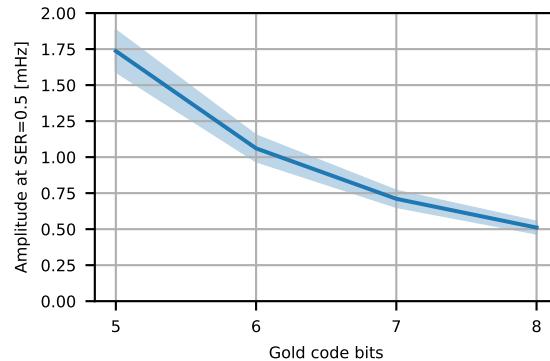
```

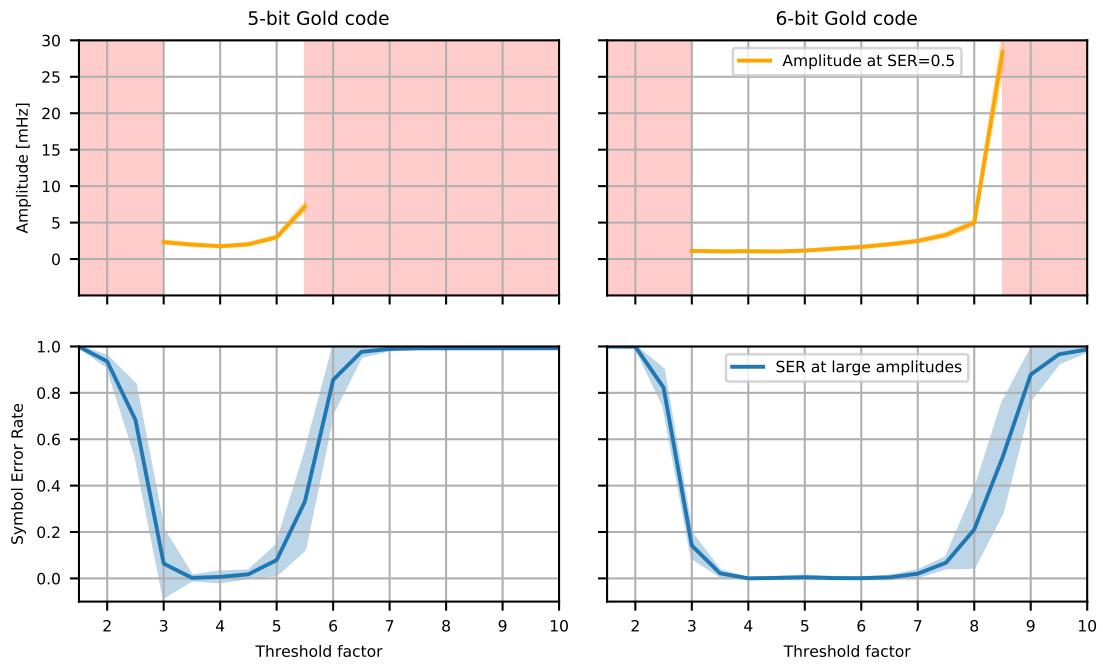
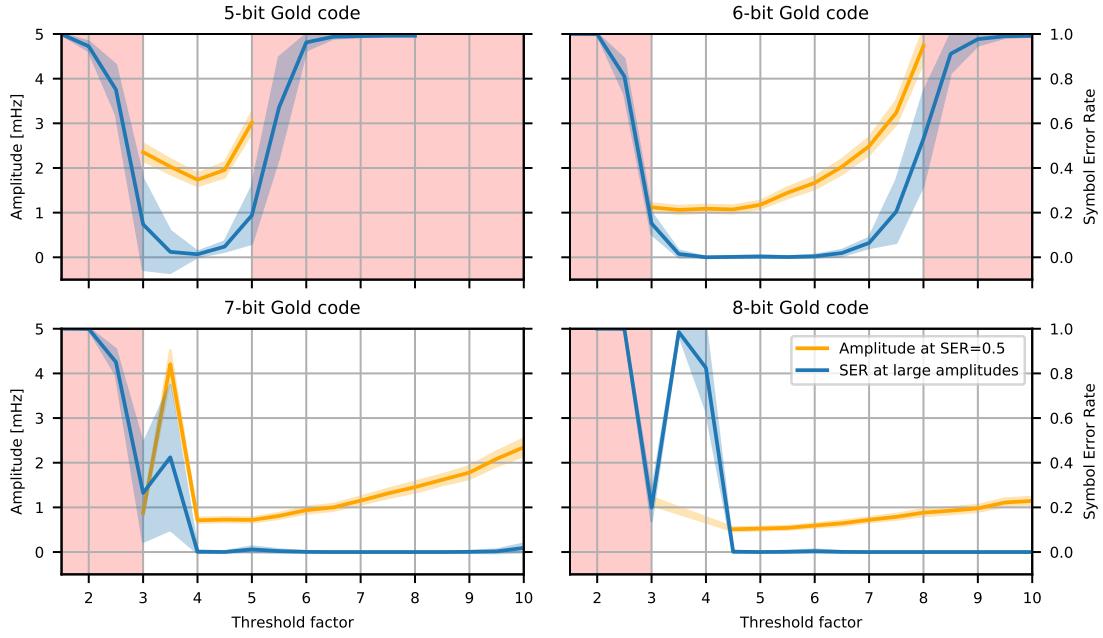
```

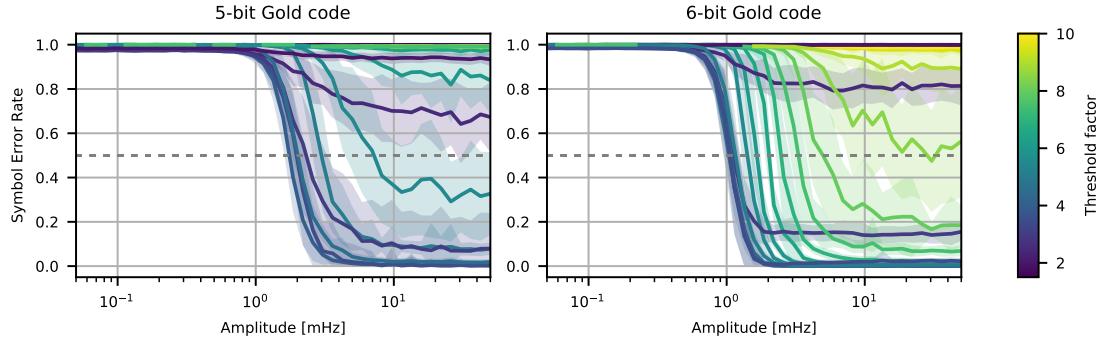
plot_thf_graph(load_results('data/
                           ↳dsss_experiments_res-par114-run120-0-2020-04-08-20-03-56.json'),
                nbitsets=[5,6], ic_ylim=[-4.99, 30],
                figsize=(7, 4))\
.savefig('fig_out/dsss_thf_sensitivity_56.pdf');

plot_amplitude_ber(load_results('data/
                                   ↳dsss_experiments_res-par114-run120-0-2020-04-08-20-03-56.json'), ↳
                     nbitsets=[5,6],
                     xlog=True, xlim=[5e-2, 5e1],
                     figsize=(7, 4))\
.savefig('fig_out/dsss_thf_amplitude_56.pdf');

```







```
[26]: def load_results_fw_sim(*files, background=None, filter_decimation=None):
    results = defaultdict(lambda: defaultdict(lambda: defaultdict(lambda: [])))

    for fn in files:
        with open(fn, 'r') as f:
            for (nbits, thf, decimation, symbols, seed, amp, background), result in json.load(f):
                if filter_decimation is None or decimation == filter_decimation:
                    results[background][(nbits, thf, symbols, decimation)][amp].append(result)

    if len(results) > 1:
        if background is None:
            raise ValueError('Results series contains series for multiple noise backgrounds. Please select one.')
        else:
            results = results[background]
    else:
        results = list(results.values())[0]

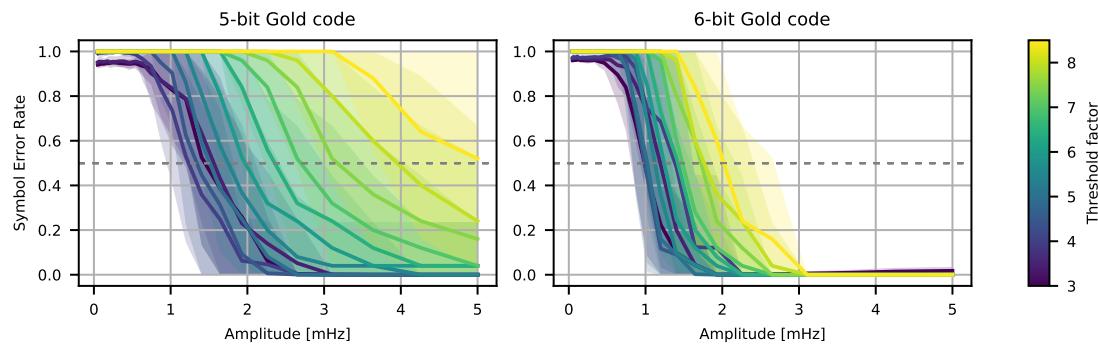
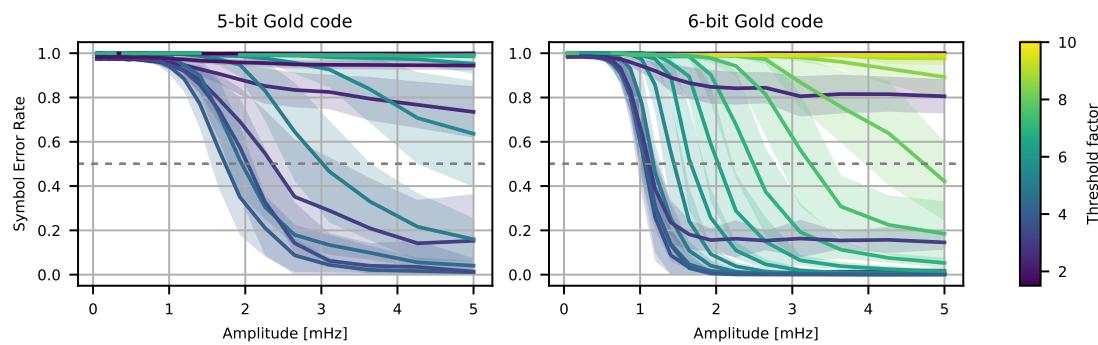
    out = []
    for (nbits, thf, duration, decimation), series in results.items():
        out_series = []
        for amplitude, amplitude_series in sorted(series.items(), key=lambda x:x[0]):
            reps = len(amplitude_series)
            out_amplitude_series = [(ser if ser is not None else 1.0, None, None) for ser in amplitude_series]
            out_series.append((amplitude, out_amplitude_series))
        out.append((nbits, thf, reps, len(series), duration, decimation), out_series)
    return out
```

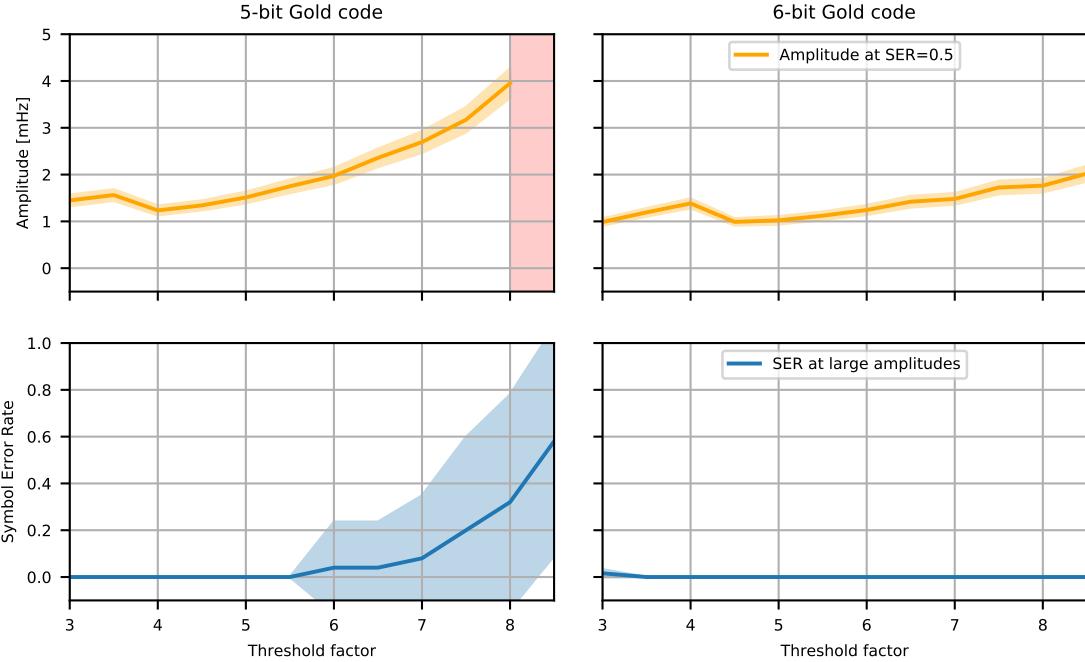
```

plot_amplitude_ber(load_results(*default_files), nbitss=[5, 6], figsize=(7, 4))\
.savefig('fig_out/dsss_thf_amplitude_56_jupyter_impl.pdf');

fw_sim_res = load_results_fw_sim(*glob.glob('data/fw_sim_ser_2/*.json'),\
filter_decimation=10)
plot_amplitude_ber(results=fw_sim_res, nbitss=[5, 6], figsize=(7, 4))\
.savefig('fig_out/dsss_thf_amplitude_56_fw_impl.pdf');
plot_thf_graph(results=fw_sim_res, nbitss=[5, 6], figsize=(7, 4))\
.savefig('fig_out/dsss_thf_sensitivity_56_fw_impl.pdf');

```



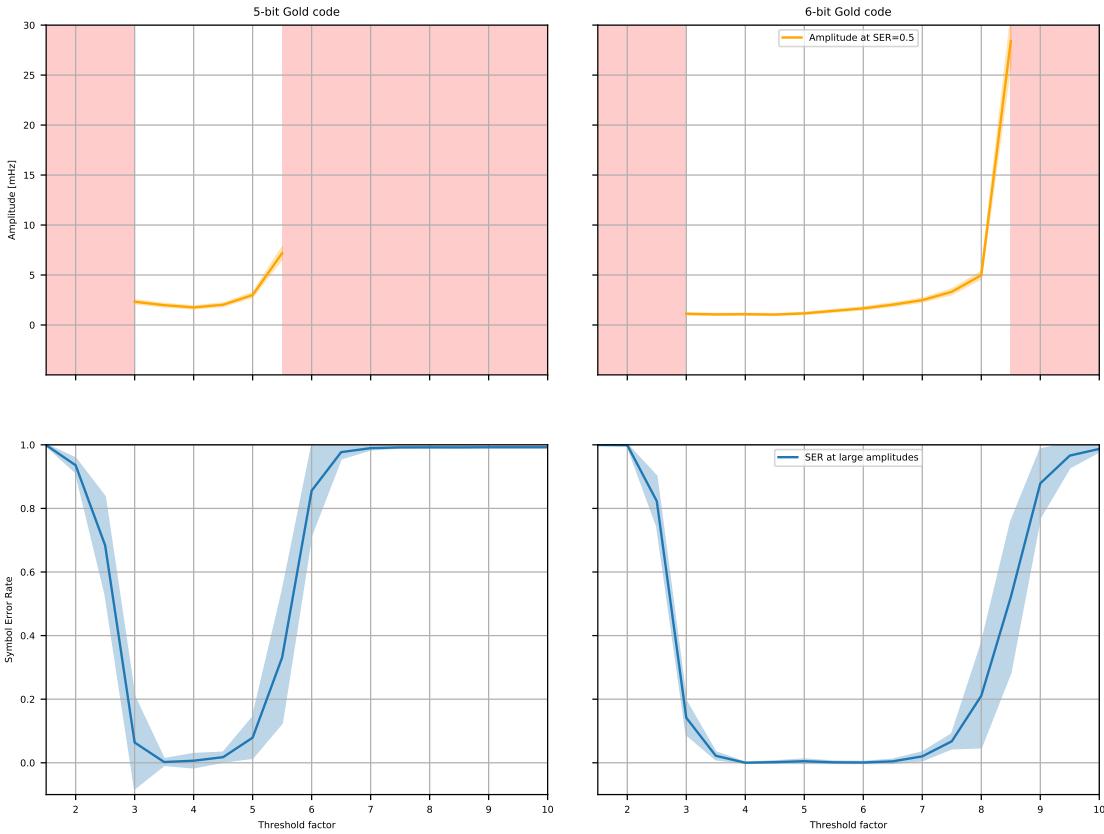


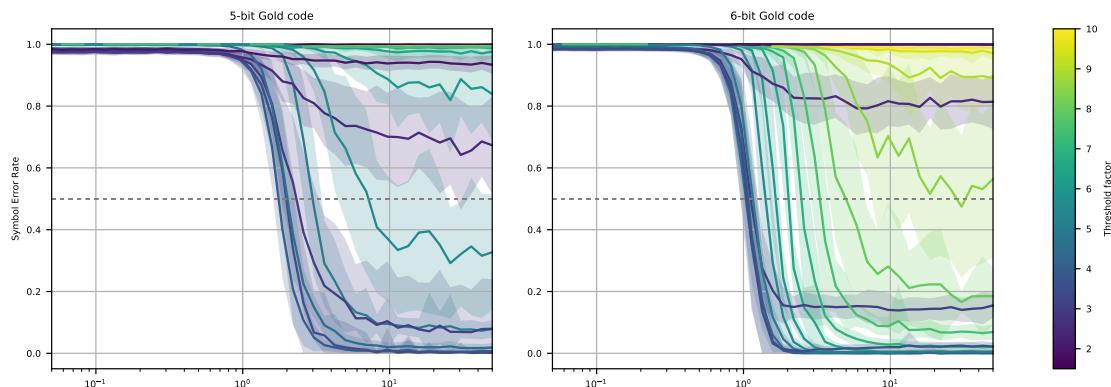
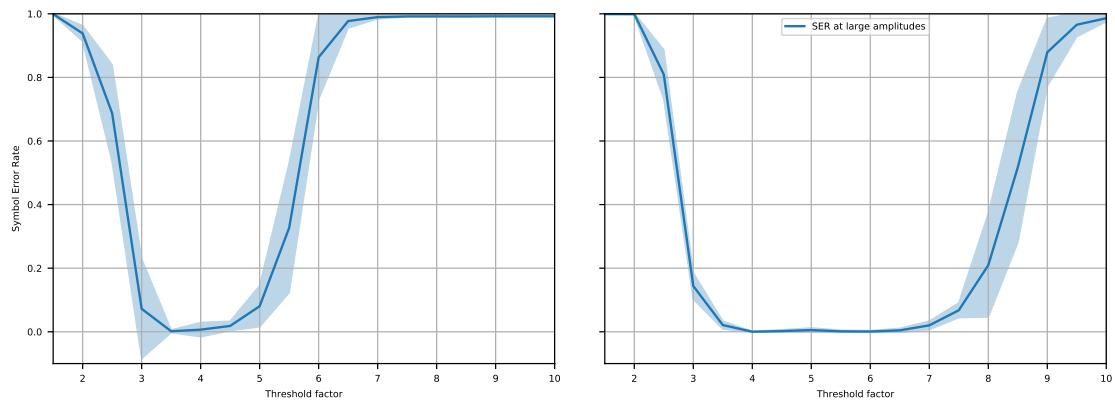
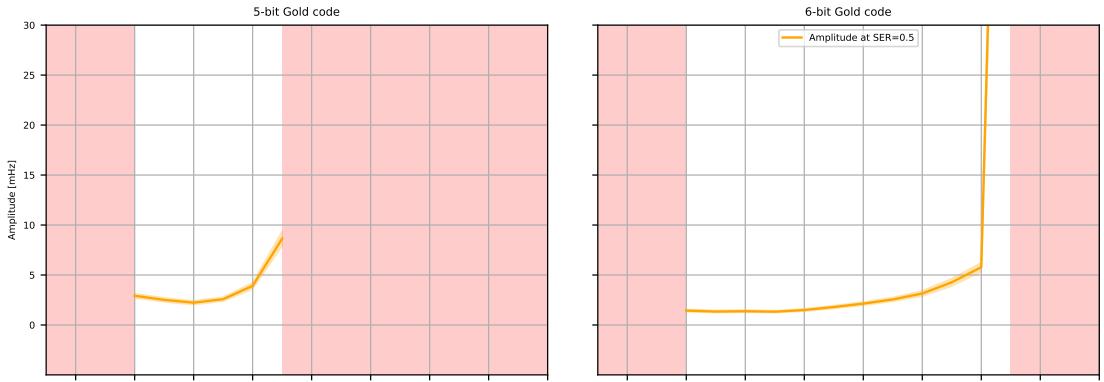
```
[70]: #sorted([x[0] for x in fw_sim_res])
#sorted({amp for _params, series in fw_sim_res for amp, reps in series}), \
#sorted({amp for _params, series in load_results(*default_files) for amp, reps in series})
```

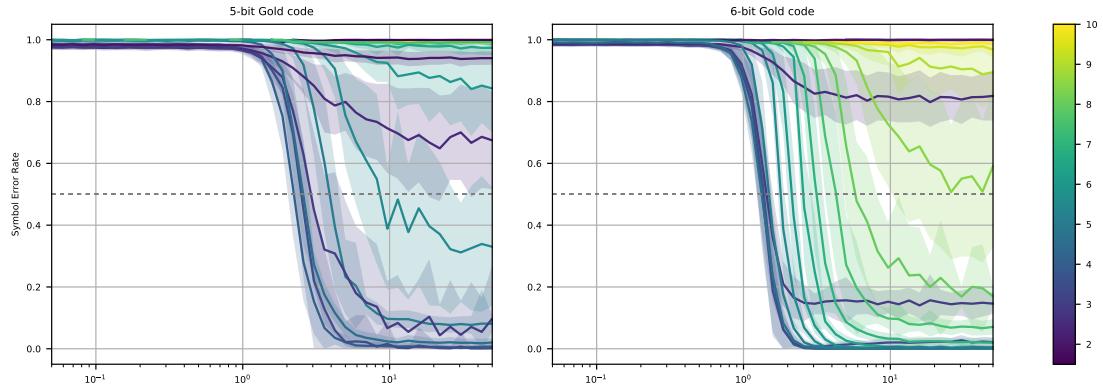
```
[43]: extra_amp_files = ['data/
˓→dsss_experiments_res-par114-run120-0-2020-04-08-20-03-56.json']
synth_files = [
    'data/dsss_experiments_res-par115-synth-run122-0-2020-04-11-20-07-33.json',
    'data/dsss_experiments_res-par115-synth-run122-1-2020-04-11-20-39-19.json'
]

# Note: due to a mistake these "par114" files actually contain "par115" data.
plot_thf_graph(extra_amp_files, nbitses=[5,6], ic_ylim=[-4.99, 30],);
plot_thf_graph(synth_files, nbiteses=[5,6], ic_ylim=[-4.99, 30]);

plot_amplitude_ber(extra_amp_files, nbiteses=[5,6], xlog=True, xlim=[5e-2, 5e1]);
plot_amplitude_ber(synth_files, nbiteses=[5,6], xlog=True, xlim=[5e-2, 5e1]);
```







```
[59]: chip_duration_default_files = [
    #   'data/dsss_experiments_res-2020-02-20-14-10-13.json',
    #   'data/dsss_experiments_res-2020-02-20-13-21-57.json',
    #   'data/dsss_experiments_res-2020-02-20-13-23-47.json',
    #   'data/dsss_experiments_res-2020-02-20-19-51-21.json',
    #   'data/dsss_experiments_res-2020-02-20-20-43-32.json',
    #   'data/dsss_experiments_res-2020-02-20-21-36-42.json',
    #   'data/dsss_experiments_res-par107-run115-0-2020-04-07-11-41-37.json',
    #   'data/dsss_experiments_res-par107-run115-1-2020-04-07-13-23-42.json',
    #   'data/dsss_experiments_res-par107-run115-2-2020-04-07-08-57-38.json',
    #   'data/dsss_experiments_res-par107-run115-3-2020-04-07-15-48-04.json',
    'data/dsss_experiments_res-par114-run119-0-2020-04-08-20-13-44.json'
]

def plot_chip_duration_sensitivity(only_nbites=5,
                                   files=chip_duration_default_files, only_thf=None, figsize=(12, 9)):
    fig, ((ax, cbar_ax), (intercept_ax, empty)) = plt.subplots(2, 2,
                                                               figsize=figsize, gridspec_kw={'width_ratios': [1, 0.05], 'hspace': 0.5})
    empty.axis('off')
    #fig.tight_layout()

    results = []

    for fn in files:
        with open(fn, 'r') as f:
            results += json.load(f)

    decimations = [decimation for (_nbites, thf, _reps, _points, _duration, _decimation), series in results if decimation > 0]
    cmap = matplotlib.cm.viridis
    cm_func = lambda x: cmap(np.log10(x - min(decimations)) / (np.log10(max(decimations)) - np.log10(min(decimations))))
```

```

decimation_sers = {}
for (nbits, thf, reps, points, duration, decimation), series in results:
    if only_thf is not None and thf != only_thf:
        continue
    if nbits != only_nbites:
        continue
    if not decimation > 0:
        continue
    data = [ [ mean for mean, _std, _msg in reps if mean is not None ] for
→_amp, reps in series ]
    amps = [ amp for amp, _reps in series ]
    sers = np.array([ np.mean(values) for values in data ])
    stds = np.array([ np.std(values) for values in data ])
    decimation_sers[decimation] = list(zip(amps, sers, stds))

    amps = [ amp*1000 for amp in amps ]
    l, = ax.plot(amps, np.clip(sers, 0, 1),_
→label=f'decimation={decimation}', color=cm_func(decimation))
    ax.fill_between(amps, np.clip(sers + stds, 0, 1), np.clip(sers - stds,_
→0, 1), facecolor=l.get_color(), alpha=0.2)
    ax.axhline(0.5, color='gray', ls=(0, (3, 4)), lw=0.8)
    ax.grid()
    ax.set_xlabel('Amplitude [mHz]')
    ax.set_ylabel('Symbol error rate')
    ax.set_title(f'{only_nbites}-bit Gold code')

    norm = matplotlib.colors.Normalize(vmin=np.log10(min(decimations)), vmax=np.
→log10(max(decimations)))
    tick_decs = sorted(set(float(dec) for dec in decimations))
    yticks = [np.log10(d) for d in tick_decs]
    cb1 = matplotlib.colorbar.ColorbarBase(cbar_ax, cmap=cmap, norm=norm,_
→orientation='vertical', ticks=yticks)
    cb1t = cbar_ax.twinx()
    cb1t.set_ylim(cbar_ax.get_ylimits())
    cb1t.set_yticks(yticks)

    cbar_ax.set_yticklabels([f'{d/sampling_rate:.1f}' for d in tick_decs])
    cbar_ax.set_ylabel("chip duration [s]", labelpad=-40)

    cb1t.set_yticklabels([f'{d/sampling_rate * 2**only_nbites:.1f}' for d in_
→tick_decs])
    cb1t.set_ylabel("symbol duration [s]")


def plot_intercepts(ax, SER_TH = 0.5):
    intercepts = {}

```

```

    for dec, sers in decimation_sers.items():
        last_ser, last_amp, last_std = 0, 0, 0
        for amp, ser, std in sorted(sers):
            if last_ser > SER_TH and ser < SER_TH:
                icp = last_amp + (SER_TH - last_ser) / (ser - last_ser) * ↴
        ↵(amp - last_amp)
                ic_std = (abs(last_amp - amp) / 2) + np.sqrt(np.
        ↵mean(last_std**2 + std**2))
                intercepts[dec] = (icp, ic_std)
                break
            last_amp, last_ser = amp, ser
        else:
            intercepts[dec] = None, None

    ser_valid = [dec for dec, (ser, _std) in intercepts.items() if ser is ↴
        ↵not None]
    #ax.axvline(min(ser_valid), color='red')
    #ax.axvline(max(ser_valid), color='red')

    x = sorted(intercepts.keys())
    data = np.array([intercepts[dec] for dec in x])
    y = data[:,0]
    std = data[:,1]
    ax.set_xlim([min(x), max(x)])
    y = [v*1000 if v is not None else v for v in y]
    l = ax.plot(x, y, label='Amplitude at SER=0.5 [mHz]', color='orange')
    #ax.legend(loc=3)
    ax.set_ylabel('Amplitude at SER=0.5 [mHz]')
    ax.grid()

    x, y, std = zip(*[(le_x, le_y, le_std) for le_x, le_y, le_std in ↴
        ↵zip(x, y, std) if le_y is not None])
    y, std = np.array(y), np.array(std)
    ax.fill_between(x, y-std, y+std, color=l[0].get_color(), alpha=0.3)

    trans = matplotlib.transforms.blended_transform_factory(ax.transData, ↴
        ↵ax.transAxes)
    ax.fill_between([-1, min(ser_valid)], 0, 1, facecolor='red', alpha=0.2, ↴
        ↵transform=trans, zorder=1)
    ax.fill_between([max(ser_valid), max(ser_valid)*10], 0, 1, ↴
        ↵facecolor='red', alpha=0.2, transform=trans)
    ax.set_ylim([min(y)*0.9, max(y)*1.1])
    ax.set_xscale('log')
    ax.xaxis.set_major_formatter(matplotlib.ticker.FuncFormatter(lambda x, ↴
        ↵_: '{:g}'.format(x)))
    xticks = [1, 2, 5, 10, 20, 50]

```

```

    ax.set_xticks(xticks)
    ax.set_xticklabels([ f'{x/sampling_rate:.1f}' for x in xticks ])
    ax.set_xlim([1, 60])
    ax.set_xlabel('chip duration [s]')

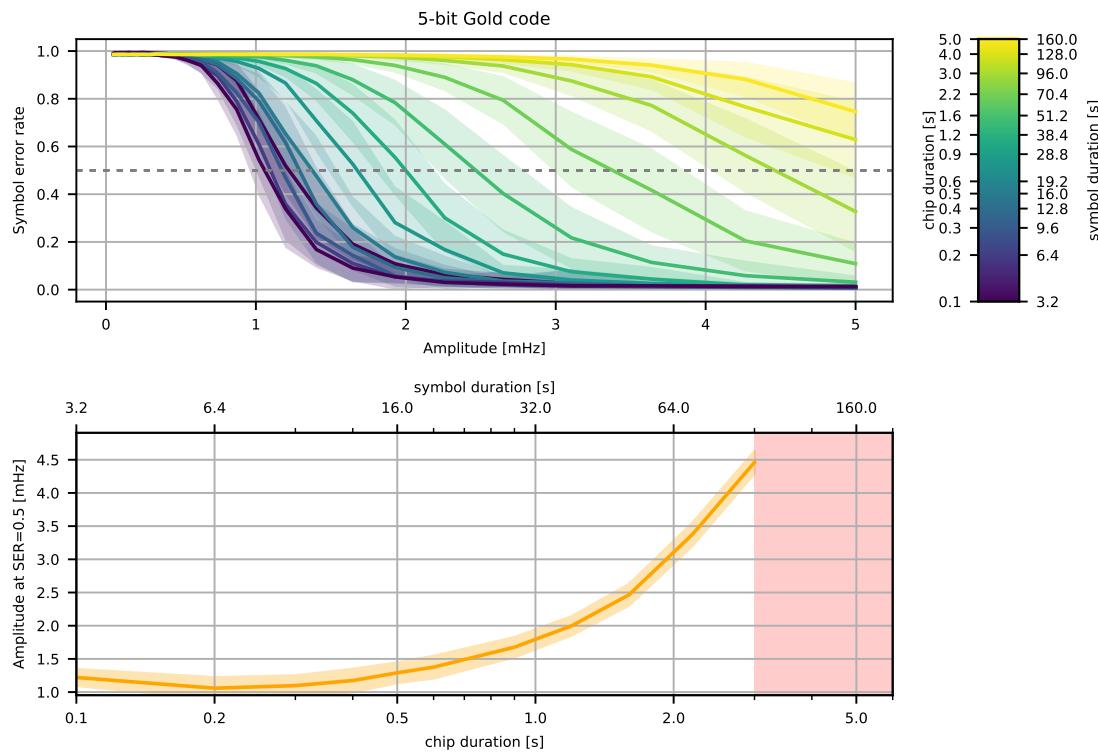
    axt = ax.twiny()
    axt.set_xlim(ax.get_xlim())
    axt.set_xscale('log')
    axt.set_xticks(xticks)
    axt.set_xticklabels([ f'{x/sampling_rate * 2**only_nbis:.1f}' for x in u
    ↪xticks ])
    axt.set_xlabel('symbol duration [s]')

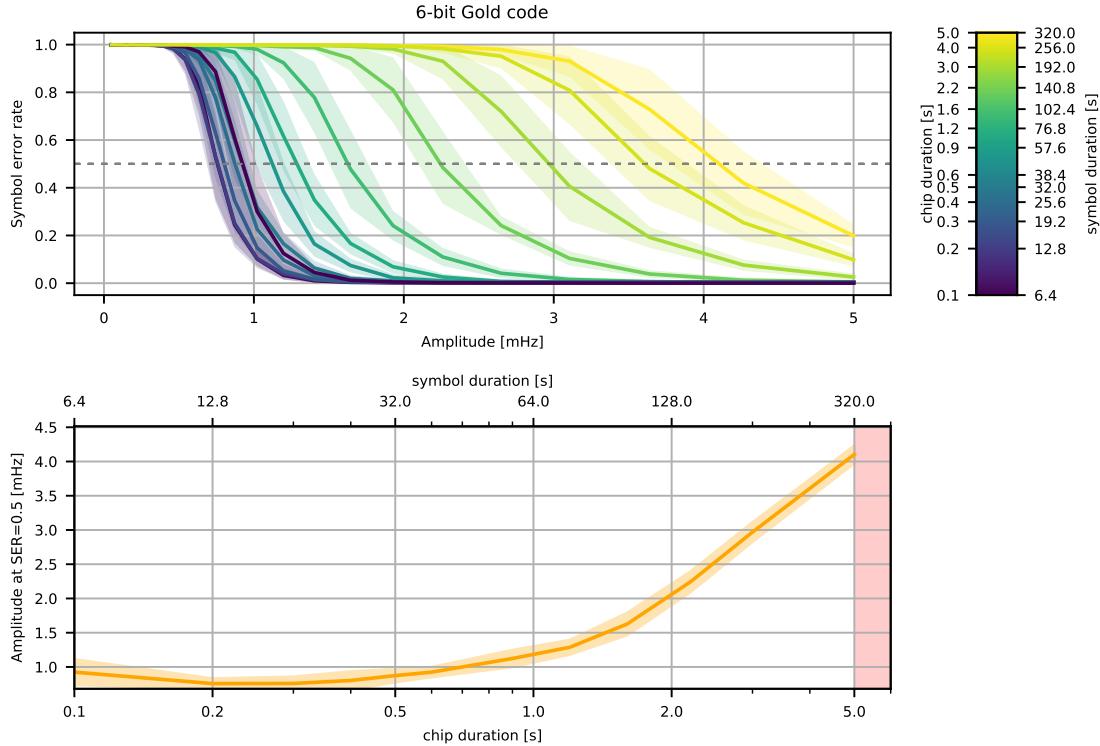
    return 1

l1 = plot_intercepts(intercept_ax)
return fig

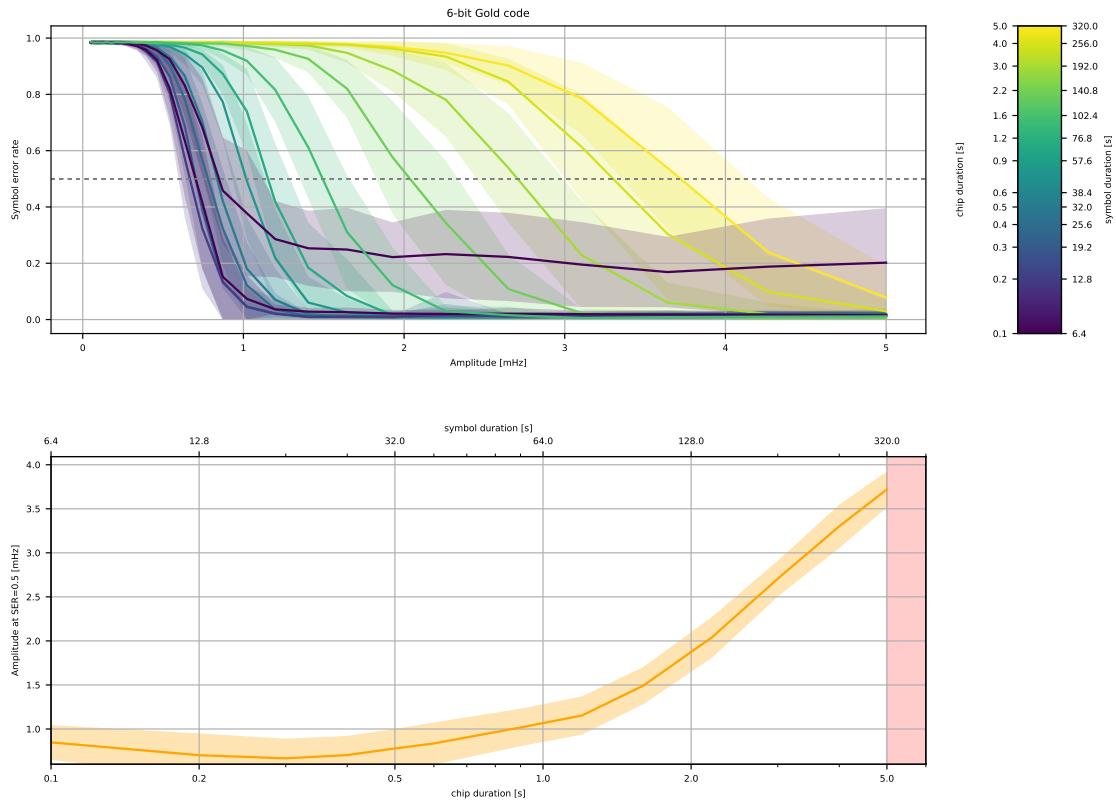
plot_chip_duration_sensitivity(5, figsize=(7, 5))\
.savefig('fig_out/chip_duration_sensitivity_5.pdf');
plot_chip_duration_sensitivity(6, figsize=(7, 5))\
.savefig('fig_out/chip_duration_sensitivity_6.pdf');

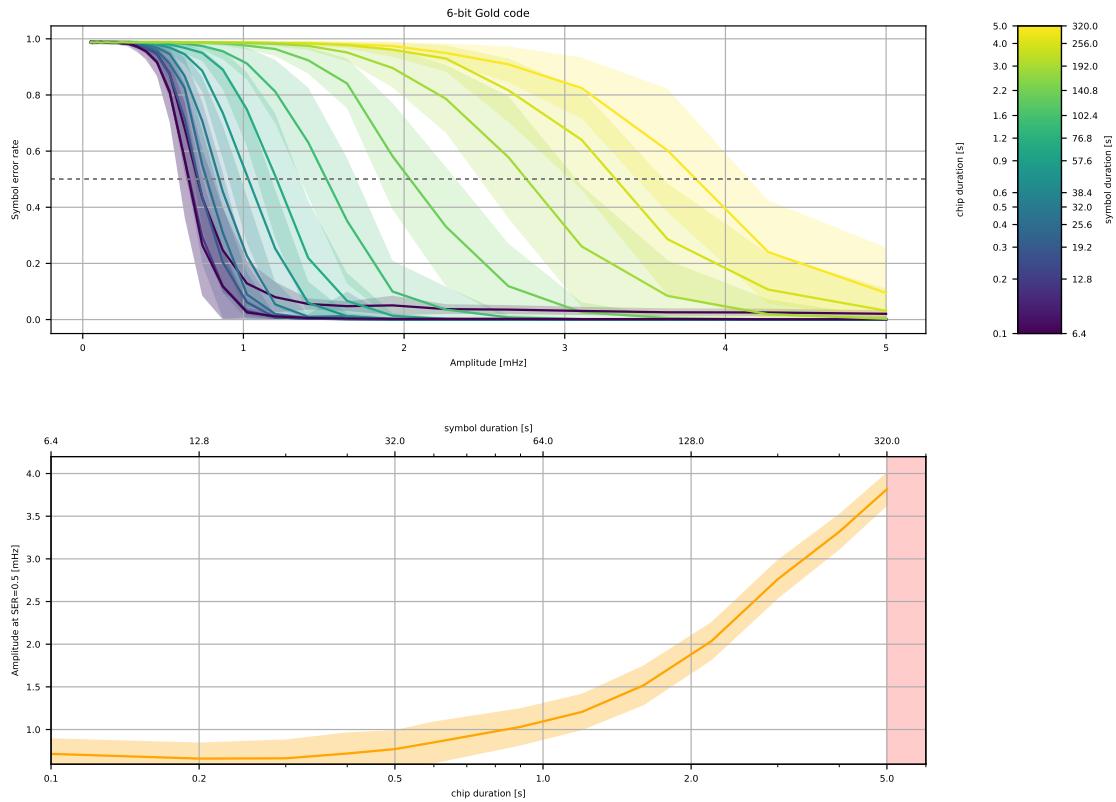
```

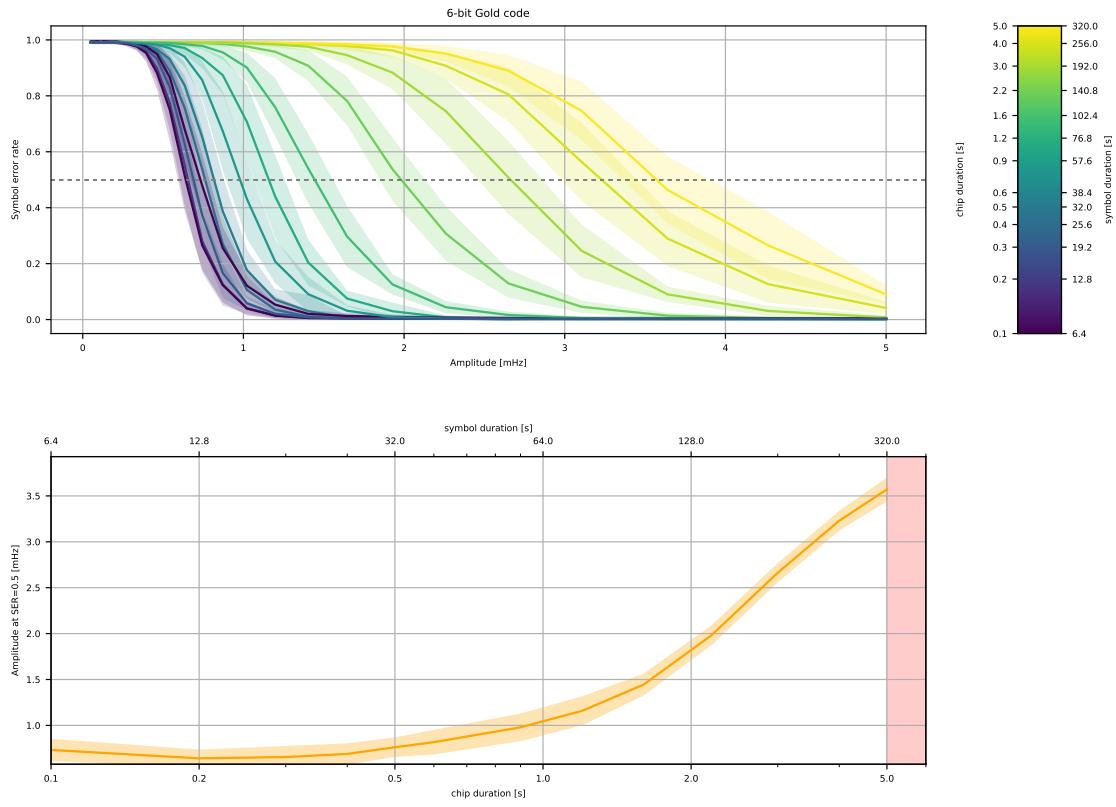


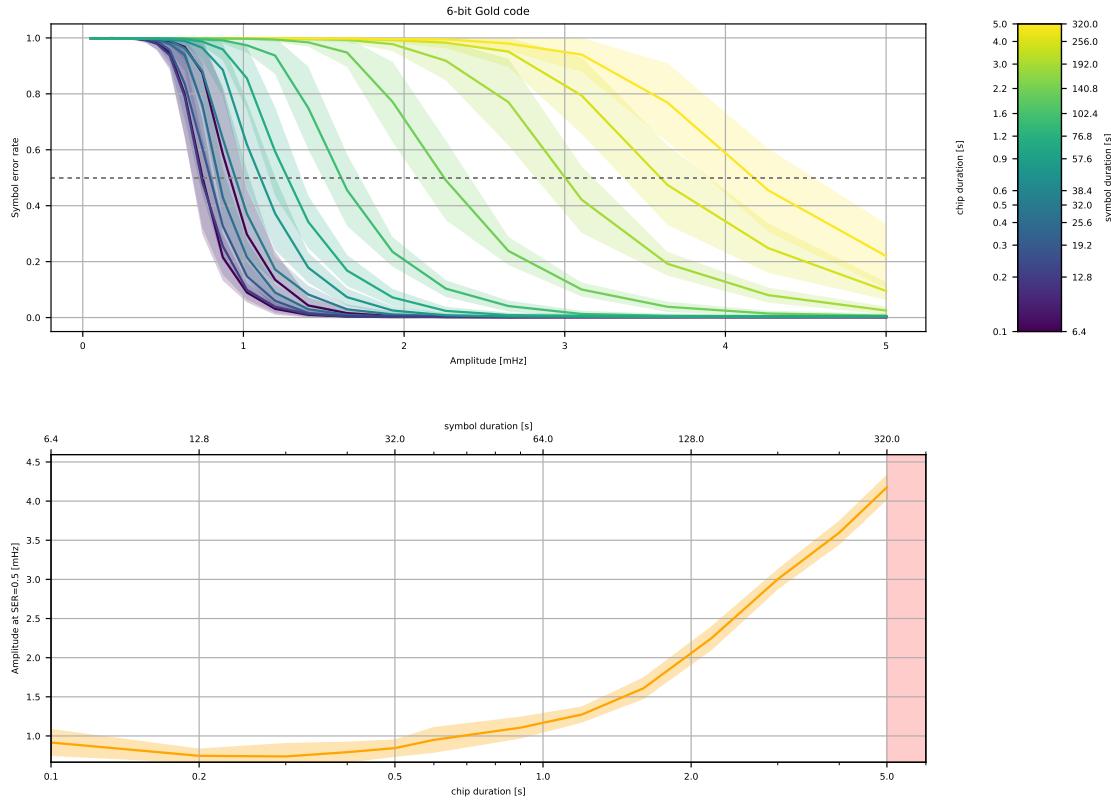


```
[45]: new_files = [
    'data/dsss_experiments_res-par111-run119-0-2020-04-09-04-02-53.json',
    'data/dsss_experiments_res-par111-run119-1-2020-04-08-16-11-20.json',
    'data/dsss_experiments_res-par111-run119-2-2020-04-08-18-07-22.json',
    'data/dsss_experiments_res-par111-run119-3-2020-04-08-13-56-03.json',
]
plot_chip_duration_sensitivity(6, only_thf=3.5, files=new_files);
plot_chip_duration_sensitivity(6, only_thf=4.0, files=new_files);
plot_chip_duration_sensitivity(6, only_thf=4.5, files=new_files);
plot_chip_duration_sensitivity(6, only_thf=5.0, files=new_files);
#plot_chip_duration_sensitivity(6, files=new_files);
```









```
[60]: synth_files = [
    'data/dsss_experiments_res-par114-synth-run121-0-2020-04-11-17-50-31.json',
    'data/dsss_experiments_res-par114-synth-run121-1-2020-04-12-03-46-19.json',
    'data/dsss_experiments_res-par114-synth-run121-2-2020-04-11-18-44-15.json',
    'data/dsss_experiments_res-par114-synth-run121-3-2020-04-11-15-25-53.json',
]
plot_chip_duration_sensitivity(5, only_thf=4.0, files=new_files);
plot_chip_duration_sensitivity(5, only_thf=4.0, files=synth_files);
plot_chip_duration_sensitivity(6, only_thf=5.0, files=new_files, figsize=(7, 5))\
    .savefig('fig_out/chip_duration_sensitivity_cmp_meas_6.pdf');
plot_chip_duration_sensitivity(6, only_thf=5.0, files=synth_files, figsize=(7, 5))\
    .savefig('fig_out/chip_duration_sensitivity_cmp_synth_6.pdf');
```

